## 1 Transforming Geometric Objects <br> Pacing: 27 Days

## Topic 1: Rigid Motion Transformations

Students use patty paper and the coordinate plane to investigate the creation of congruent figures with translations, reflections, and rotations. They use patty paper to develop intuition about the properties of the transformations, and then make generalizations about the coordinates of figures after transformations. Students determine sequences of transformations that map congruent figures to each other.

| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Patty Paper, Patty Paper <br> Introduction to Congruent Figures | $\begin{aligned} & \text { 8.G.1.a } \\ & \text { 8.G.1.b } \\ & \text { 8.G.2 } \end{aligned}$ | 1 | Students use patty paper to investigate congruent figures. They make conjectures about congruence, investigate their conjectures, and justify their conjectures using informal transformation language. | - Patty paper is a hands-on tool for investigating geometric ideas. <br> - If two figures are congruent figures, all corresponding sides and all corresponding angles have the same measure. <br> - Corresponding sides are sides that have the same relative position in geometric figures. <br> - Corresponding angles are angles that have the same relative position in geometric figures. <br> - A rigorous study of geometry requires making conjectures, investigating conjectures, and justifying true results. |
| 2 | Slides, Flips, and Spins <br> Introduction to Rigid Motions | $\begin{aligned} & \text { 8.G.1.a } \\ & \text { 8.G.1.b } \\ & \text { 8.G.1.c } \\ & \text { 8.G.2 } \end{aligned}$ | 3 | Students develop a formal understanding of translations, rotations, and reflections in the plane. They use patty paper to investigate each transformation, create images from pre-images, and determine the properties of each transformation. They learn that each rigid motion transformation preserves the size and shape of the original figure, and that translations and rotations also preserve the orientation of the figure. | - Transformations are mappings of a plane and all the points of a figure in a plane according to a common action or operation. <br> - Rigid motions are transformations that preserve the size and shape of a figure. <br> - Translations, rotations, and reflections are rigid motions. <br> - The pre-image of a figure is an original figure prior to a transformation. The image is the result of a transformation. <br> - Translations are "slides" of a figure in a given direction by a specific distance. <br> - Reflections are "flips" of a figure over a line of reflection. <br> - Rotations are "turns" of a figure given a center of rotation, by an angle of rotation, in a given direction. <br> - Translations, rotations, and reflections preserve segment length, angle measure, and parallelism of segments. |
| 3 | Lateral Moves <br> Translations of Figures on the Coordinate Plane | $\begin{aligned} & \text { 8.G. } 2 \\ & \text { 8.G. } 3 \end{aligned}$ | 2 | Students build on their patty paper exploration of slides, or translations. They investigate translations on the coordinate plane, conjecturing about the coordinates of images having been translated. At the end, students reason that figures that undergo translations (pre-images) are congruent to the resulting images. | - A translation is a transformation that moves each point of a figure the same distance and direction. <br> - A point with the coordinate $(x, y)$, when translated horizontally by $c$ units, has new coordinates ( $x+c, y$ ). A point with the coordinates ( $x, y$ ), when translated vertically by $c$ units, has new coordinates $(x, y+c)$. |


| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
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## Topic 2: Similarity

Students investigate dilations and similarity. They make connections between scale factors and dilation factors and define similar figures. Students dilate figures on the coordinate plane using different locations for the center of dilation, and generalize the coordinates of images formed from a dilation with a center at the origin. Then they identify a sequence of transformations that map from a figure to a similar figure.

| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Pinch-Zoom Geometry <br> Dilations of Figures | 8.G. 4 | 2 | Students review terms associated with scale factors and relate them to dilations and similarity. They learn that dilations create similar figures, with congruent corresponding angles and equal ratios of their corresponding side lengths. | - Dilation is a transformation that produces images that are the same shape as the preimage, but not the same size. <br> - When a figure is dilated with a scale factor greater than 1 , the resulting figure is a similar figure that is an enlargement because each side length is multiplied by a scale factor that is larger than the identity factor of 1. <br> - When a figure is dilated with a scale factor less than 1 , the resulting figure is a similar figure that is a reduction because each side length is multiplied by a scale factor that is smaller than the identity factor of 1 . |
| 2 | Rising, Running, Stepping, Scaling <br> Dilating Figures on the Coordinate Plane | $\begin{aligned} & \text { 8.G. } 3 \\ & \text { 8.G. } 4 \end{aligned}$ | 3 | Students investigate dilations on the coordinate plane, using different locations of the center of dilation. They create and modify conjectures about the effect of dilations from different centers on the coordinates of a figure. Students transformations to verify that two figures are similar. | - Dilation is a transformation that produces images that are the same shape as the preimage, but not the same size. <br> - Each of the coordinates of the points of a figure is multiplied by the scale factor when the figure is dilated using the origin as the center of dilation. |
| 3 | From Here to There <br> Mapping Similar <br> Figures using <br> Transformations | 8.G. 4 | 2 | Students determine similarity using a single dilation, and verify similarity through a sequence of transformations. They explore the relationship between images of a common pre-image under different conditions and the relationship between figures similar to congruent figures. | - If two figures are similar, one can be mapped to the other through a sequence of transformations. <br> - Images created from the same pre-image are similar. |
| Learnin MATHia | Individually with or Skills Practice | $\begin{aligned} & \text { 8.G. } 2 \\ & \text { 8.G. } 3 \\ & \text { 8.G. } \end{aligned}$ | 1 | Students dilate triangle given a center of dilation and a scale factor and describe the dilations needed to map a pre-image onto an image. Students describe the transformations needed to map a pre-image onto an image. <br> MATHia Unit: Similar Figures on the Coordinate Plane <br> MATHia Workspaces: Defining Similarity / Dilating Plane Figures / Performing One Transformation / Performing Multiple Transformations / Describing Transformations Using Coordinates |  |

## Topic 3: Line and Angle Relationships


 related problems, including ones with complex diagrams.

Standard: 8.G. 5 Pacing: 6 Days


## 2 Developing Functional Foundations

Pacing: 52 Days

## Topic 1: From Proportions to Linear Relationships


 derive the equation $y=m x+b$ for a line that passes through the point $0, b$ rather than the origin.

| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Post-Secondary Proportions <br> Representations of Proportional Relationships | 8.EE. 5 | 2 | Students use ratios to analyze proportional relationship representations. They write equivalent ratios, write equations, create a table of values, and graph proportional relationships. Then, students analyze two scenarios involving proportional relationships represented in different forms. | - A proportional relationship is one in which the ratio of two quantities is constant. <br> - A proportional relationship is represented as a linear graph passing through the origin. <br> - A proportional relationship is represented as a table with the values that increase or decrease at a constant rate beginning or ending with ( 0,0 ). <br> - The equation for a proportional relationship is written in the form $y=k x$, where $k$ is the constant of proportionality. |
| 2 | Jack and Jill Went Up the Hill <br> Using Similar Triangles to Describe the Steepness of a Line | 8.EE. 5 <br> 8.EE. 6 | 3 | Students connect unit rate, constant of proportionality, and scale factor with slope, which is introduced as the rate of change of the dependent quantity compared to the independent quantity. Students derive the equations for a proportional linear relationship, $y=m x$, and for a non-proportional linear relationship, $y=m x+b$. | - A rate of change is used to describe the rate of increase or decrease of one quantity relative to another quantity. <br> - A unit rate is a comparison of two measurements in which the denominator has a value of one unit. <br> - The rate of change is the same for any two points on a line. <br> - An increasing line has a positive slope; a decreasing line has a negative slope. |
| 3 | Slippery Slopes <br> Exploring Slopes using <br> Similar Triangles | 8.EE. 6 | 2 | Students use similar triangles to explain why the slope of any line is the same between any two distinct points on a non-vertical line. Students explore the slope of $y=x$ and $y=2 x-5$ by drawing right triangles on the lines, verifying that the triangles are similar, and show the slope of any two points on the non-vertical line is the same. | - The slope is the same for any two points on a line. <br> - In a table or graph representing a linear relationship, the rate of change is the change in the dependent quantity divided by the corresponding change in the independent quantity (riserun). <br> - In a linear equation written in the form $y=m x+b$, the coefficient $m$ is the slope. <br> - The properties of similar triangles constructed on a line can explain why the slope is the same for any two points on a line. |


| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | Up, Down, and All Around <br> Transformations of Lines | $\begin{aligned} & \text { 8.EE.6 } \\ & \text { 8.G.1.a } \\ & \text { 8.G.1.c } \end{aligned}$ | 2 | Students apply geometric transformations to the basic function, $y=x$. They recognize $y=m x$ as a dilation of $y=x$ and $y=m x+b$ as a translation of $y=m x$. Students learn that lines with the same slope are parallel and that parallel lines remain parallel after a reflection or rotation. | - When a linear equation is dilated by a non-zero factor other than 1 , the slope of the line changes by the factor. <br> - When a linear function is translated horizontally or vertically, the slope remains the same but the intercepts change. <br> - Linear functions can be graphed using transformations. <br> - When lines are parallel to each other, the slope values in their equations are equal. <br> - Translating a line results in a line parallel to the original line. <br> - Rotating parallel lines results in parallel lines. <br> - Reflecting parallel lines results in parallel lines. |
| Learnin MATHia | Individually with or Skills Practice | 8.F. 4 | 3 | Students determine linear expressions that represent real-world context. They use these expressions to solve problems. <br> MATHia Unit: Representing Proportional Relationships <br> MATHia Workspace: Representing Proportional Relationships Algebraically / Modeling the Constant of Proportionality / Understanding the Slopes of Lines / Graphing Linear Relationships |  |

## Topic 2: Linear Relationships


 equation, including standard form.

| quation, |  |  |  |  | Essential Ideas $\quad$ Standard: 8.F.4 Pacing:19 Days |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary |  |
| 1 | U.S. Shirts <br> Using Tables, Graphs, and Equations | 8.F. 4 | 2 | Students create equations, tables, and graphs to analyze linear relationships. They compare two t-shirt company pricing rates algebraically and graphically. They then write a response that compares the pricing plans for the two companies and predict how the pricing affects business. | - Two linear relationships can be compared algebraically. <br> - Two linear relationships can be compared graphically. |
| 2 | At the Arcade <br> Linear Relationships in Tables | 8.F. 4 | 2 | Students use the formula $\left(y_{2}-y_{1}\right) /\left(x_{2}-x_{1}\right)$ to determine the rate of change for a table of values (or two points). They determine whether a given table of values represents a linear relationship, including the by examining first differences. | - The formula $\left(y_{2}-y_{1}\right) /\left(x_{2}-x_{1}\right)$, where the first point is $\left(x_{1}, y_{1}\right)$ and the second point is $\left(x_{2}, y_{2}\right)$ can be used to calculate the rate of change of a linear relationship from a table of values or two coordinate pairs. <br> - If the rate of change between consecutive ordered pairs in a table is the same value every time, the table of values represents a linear relation. <br> - First differences are the values determined by subtracting consecutive $y$-values in a table when the $x$-values are consecutive integers. If the first differences are the same every time, the table of values represents a linear relation. |
| 3 | Dining, Dancing, and Driving <br> Linear Relationships in Contexts | 8.F. 4 | 1 | Students analyze a context that represents linear relationships among distance, cost, and gallons of gas. They represent the same context using different independent and dependent quantities, each time calculating the rate in order to connect processes and representations. | - A context representing a linear function often provides enough information to determine the rate of change of the function. <br> - There are similarities in the processes of determining the rate of change from a context, graph, and table. <br> - In order to calculate a unit rate from a context representing a linear function, a rate or two data points must be provided. |
| 4 | Derby Day <br> Slope-Intercept Form of a Line | 8.F. 4 | 2 | Students learn about the y-intercept of a linear graph and the slopeintercept form of a linear equation. They use the slope formula in their calculations and to derive the slopeintercept form. Students practice writing equations in slope-intercept form. | - The $y$-intercept is the $y$-coordinate of the point where a graph crosses the $y$-axis. <br> - The $y$-intercept is written as the coordinate pair $(0, y)$. <br> - The $y$-intercept can be determined from any representation (context, table, graph, or equation) of a linear relationship. <br> - In cases where the $y$-intercept of a linear equation is not obvious, it is helpful to use the slope and algebra to calculate its value. <br> - The slope-intercept form of a linear equation is $y=m x+b$, where $m$ is the slope of the line and $(0, b)$ is the $y$-intercept of the line. |


| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
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| 5 | What's the Point? <br> Point-Slope Form of a Line | 8.F. 4 | 2 | Students use the slope formula to derive the point-slope form of a linear equation. They practice writing equations in point-slope and slope-intercept forms when given a variety of information. Students write equations for horizontal and vertical lines. | - The slope-intercept form of a linear equation is $y=m x+b$, where $m$ is the slope of the line and $(0, b)$ is the $y$-intercept of the line. <br> - The point-slope form of a linear equation is $y-y 1=m(x-x 1)$, where $m$ is the slope of the line and ( $x 1, y 1$ ) is a point on the line. <br> - Horizontal lines have a slope of 0 . The equation of a horizontal line that passes through $(0, a)$ is $y=a$. <br> - Vertical lines have an undefined slope. The equation of a vertical line that passes through $(b, 0)$ is $x=b$. |
| 6 | The Arts Are Alive Using Linear Equations | 8.F. 4 | 2 | Students graph lines using each of the three forms of linear equations: slope-intercept, point-slope, and standard form. They compare the advantages and disadvantages of each form. | - The slope-intercept form of a linear equation is $y=m x+b$, where $m$ is the slope of the line and $(0, b)$ is the $y$-intercept of the line. <br> - The point-slope form of a linear equation is $y-y 1=m(x-x 1)$, where $m$ is the slope of the line and $(x 1, y 1)$ is a point on the line. <br> - The standard form of a linear equation is $A x+B y=C$, where $A, B$, and $C$ are constants and $A$ and $B$ are not both zero. <br> - The information contained in the equation of a line can be used to graph the line. |
| Learning MATHia | Individually with or Skills Practice | 8.F. 4 | 8 | Students determine the slope of a line from a graph. Students write equations given a scenario, two points, or the slope and $y$-intercept. Students model scenarios using equations, tables, and graphs. <br> MATHia Unit: Linear Models <br> MATHia Workspaces: Multiple Representations of Linear Functions / Modeling Linear Functions Using Multiple Representations / Calculating Slope <br> MATHia Unit: Writing Equations of a Line <br> MATHia Workspaces: Connecting Slope-Intercept and Point-Slope Forms / Writing Equations Given Slope and a Point / Writing Equations Given Two Points / Modeling Linear Relationships Given Two Points / Modeling Linear Relationships Given an Initial Point / Modeling Linear\Relationships Given Two Points <br> MATHia Unit: Graphs of Linear Equations in Two Variables <br> MATHia Workspaces: Analyzing Models of Linear Relationships / Graphing Given an Integer Slope and y-Intercept / Graphing Given a Decimal Slope and $y$-Intercept / Modeling Linear Equations in Standard Form / Graphing Linear Equations using a Given Method / Graphing Linear Equations using a Chosen Method |  |

## Topic 3: Introduction to Functions



 of graphs of linear functions, and equations of linear functions as they transform and compare linear functions with either the same slope.

Standards: 8.F.1, 8.F.2, 8.F.3, 8.F.4, 8.F.5, 8.G.1.a, 8.G.1.c Pacing: 12 Days

| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Patterns, Sequences, Rules ... <br> Analyzing Sequences as Rules | 8.F. 1 | 1 | Students analyze sequences that decrease, increase, and alternate between decreasing and increasing. They generate the next terms, describe the patterns in the sequences, and compare the sequences. | - A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A term in a sequence is an individual number, figure, or letter in the sequence. <br> - There are many different patterns that can generate a sequence of numbers. Some possible patterns are: <br> - Adding or subtracting by the same number each time. <br> - Multiplying or dividing by the same number each time. <br> - Adding by a different number each time, with the numbers being part of a pattern. <br> - Alternating between adding and subtracting |
| 2 | Once Upon a Graph <br> Analyzing the <br> Characteristics of Graphs of Relationships | 8.F. 5 | 2 | Students complete a sorting activity to distinguish the graphs that are discrete or continuous; linear or nonlinear; increasing, decreasing, neither increasing nor decreasing, or both increasing and decreasing. They use these terms to qualitatively describe numberless piecewise linear graphs. | - Graphs can be described by characteristics such as: discrete or continuous, linear or nonlinear and increasing, decreasing, neither increasing or decreasing, or both increasing and decreasing. <br> - A discrete graph is a graph of isolated points. A continuous graph is a graph with no breaks in it. <br> - A linear graph is a graph that is a line or series of collinear points. A nonlinear graph is a graph that is not a line and therefore not a series of collinear points. <br> - The graphs of all sequences are discrete graphs. |
| 3 | One or More Xs to One Y <br> Defining Functional Relationships | 8.F. 1 | 3 | The terms relation and function are defined. Students analyze mappings, sets of ordered pairs, sequences, tables, graphs, equations, and contexts and determine which are functions. | - A relation is any set of ordered pairs or the mapping between a set of inputs and a set of outputs. The first coordinate in an ordered pair in a relation is the input, and the second coordinate is the output. <br> - A function is a relation which maps each input to one and only one output. Relations that are not functions will have more than one output for each input. <br> - A scatter plot is a graph of a collection of ordered pairs that allows an exploration of the relationship between the points. <br> - The vertical line test is a visual method of determining whether a relation represented as a graph is a function. |


| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
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| 4 | Over the River and Through the Woods <br> Describing Functions | $\begin{aligned} & \text { 8.F. } 3 \\ & \text { 8.F. } 4 \\ & \text { 8.F. } 5 \end{aligned}$ | 2 | Students analyze the graphical behavior of linear and nonlinear functions. They conclude that nonvertical linear relationships are linear functions. Then students learn about and apply intervals of increase and decrease and constant intervals to descriptions of a day and to specific nonlinear functions (absolute value, quadratic, cubic). | - A linear function is a function whose graph is a straight line. <br> - An increasing function is when both values of the function increase. <br> - A constant function is when the $y$-value does not change or remains constant. <br> - A decreasing function is when the value of the dependent variable decreases as the independent variable increases. <br> - A function has an interval of increase when it is increasing for some values of the independent variable. <br> - A function has an interval of decrease when it is decreasing for some values of the independent variable. <br> - A function has a constant interval when it is constant for some values of the independent variable. <br> - Nonlinear functions such as absolute value functions, quadratic functions, and cubic functions can be represented with equations, tables, and graphs. |
| 5 | Comparing Apples to Oranges <br> Comparing Functions using Different Representations | 8.F. 2 | 1 | Students compare the rate of change associated with functions represented by equations, tables of values, graphs, and verbal descriptions. They also order rates of change associated with various representations. | - Functions may be represented and compared using graphs, equations, verbal descriptions, and a table of values. <br> - The properties of functions can be compared even when the functions are represented in different ways. <br> - An increasing line has a positive rate of change, a decreasing line has a negative rate of change, a horizontal line has a rate of change equal to zero, and a vertical line has a rate of change that is undefined. <br> - When comparing lines on the same graph, as the absolute value of the slope increases, the line becomes steeper, closer to vertical. As the absolute value of the slope decreases, the line becomes closer to horizontal. |
| Learni MATHi | Individually with or Skills Practice | $\begin{aligned} & \text { 8.F. } 1 \\ & \text { 8.F. } 5 \end{aligned}$ | 3 | Students write equations to represent a function given a table or graph. Students classify given relations as function or nonfunctions. Students identify key characteristics from the graph of a function. <br> MATHia Unit: Relations and Functions MATHia Workspaces: Exploring Functions / Exploring Graphs of Functions / Classifying Relations and Functions / Identifying Key Characteristics of Graphs of Functions |  |

## Topic 4: Patterns in Bivariate Data

Students investigate associations in bivariate data, both quantitative and categorical. They construct scatter plots and determine if scatter plots exhibit linear relationships and describe other patterns of association, including clustering, outliers, or a positive or negative association. Students informally fit lines of best fit, determine the equations of those lines, interpret the slopes and $y$-intercepts of the lines, and use the equations to make and judge the reasonableness of predictions about the data. Finally, students construct and interpret two-way frequency and relative frequency tables to describe possible associations between two categorical variables.

Standards: 8.SP.1, 8.SP.2, 8.SP.3, 8.SP. 4 Pacing: 9 Days

| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Pass the Squeeze <br> Analyzing Patterns in Scatter Plots | 8.SP. 1 | 2 | Students construct and analyze scatter plots of bivariate data to explore patterns in the data. They analyze scatter plots as they learn the terms bivariate data, explanatory variable, response variable, linear association, positive association, negative association, cluster, and outlier. |  |

## Essential Ideas

- Bivariate data is used when collecting information regarding two characteristics for the same person, thing, or event.
- A scatter plot is a graph of a set of ordered pairs. The points in a scatter plot are not connected.
- Scatter plots allow us to investigate patterns in bivariate data.
- In a scatter plot, if the response variable increases as the explanatory variable increases, then the two variables are said to have a positive association. If the response variable decreases as the explanatory variable increases, then the two variables are said to have a negative association.
- Common patterns in bivariate data are clustering, positive or negative associations, linear and nonlinear associations, and outliers.

Students analyze two scatter plots to show percent of book sales from bookstores and the internet for time from 2004 to 2010. They write an equation of the line of best fit for each scatter plot and draw it on their plot. Using their line of best fit, students predict the percent of book sales.

Students create a scatter plot for age and height and a scatter plot for age and weight. They draw the line of best fit and determine the equation of the line of best fit for each scatter plot. Students then make predictions for height and for weight based on age using the equation of each line of best fit.

- Outliers are points that deviate from the overall pattern of the data.
- A line of best fit is a straight line that is as close to as many points as possible, but does not have to go through any of the points on the scatter plot.
- Equations can be written for a line of best fit.
- A line of best fit can be used to make predictions about bivariate data
- A line of best fit and its equation are often referred to as a model of the data.
- A line of best fit is a straight line that is as close to as many points as possible, but does not have to go through any of the points on the scatter plot.
- Equations can be written for a line of best fit.
- A line of best fit can be used to make predictions about bivariate data.
- A line of best fit and its equation are often referred to as a model of the data.
- The closer the data points are to a line, the better the fit of the line to the data.
- You can interpret the slope and $y$-intercept of a line of best fit by looking at the problem situation and the independent and dependent quantities.

| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | The Stroop Test <br> Comparing Slopes and Intercepts of Data from Experiments | 8.SP. 3 | 1 | Students conduct the Stroop Test experiment, calculate the mean time for various matching and nonmatching lists of words, and create scatter plots of the list length versus the amount of time. They then draw the line of best fit for each scatter plot and make predictions for the amount of time based on the list length using the equations of the lines of best fit. | - Equations can be written for a line of best fit. <br> - A line of best fit can be used to make predictions about data. |
| 5 | Would You Rather ... ? <br> Patterns of Association in Two-Way Tables | 8.SP. 4 | 2 | Students construct and use twoway tables containing tally marks and numerical data describing the frequencies of occurrence. Then they construct relative frequency tables and use the tables to answer questions related to the problem situation. | - Data that is categorical or qualitative is data that is not considered quantitative or numerical. <br> - Two-way tables are used to display categorical data that shows the number of data points that fall into each group for two variables. Tally marks are often used in the tables to ensure each variable of a data point is recorded. <br> - The frequency of a variable is the number of times it appears in the data set. <br> - A relative frequency is the ratio or percent of occurrences within a category to the total of the category. <br> - Frequencies and relative frequencies in two-way tables for two-variable categorical data are used to represent relationships between two variables and to draw conclusions about situations. |
| Learnin MATHia | Individually with or Skills Practice | $\begin{aligned} & 8 . S P .1 \\ & \text { 8.SP. } 2 \\ & \text { 8.SP. } \end{aligned}$ | 1 | Students determine whether a graph shows a positive linear association, a negative linear association, a non-linear association, or no association. Students estimate and write equations for lines of best fit based on a scatter plot. Students use equations for trend lines to make predictions. <br> MATHia Unit: Lines of Best Fit <br> MATHia Workspaces: Estimating Lines of Best Fit / Using Lines of Best Fit <br> MATHia Unit: Categorical Data <br> MATHia Workspaces: Building Marginal Frequency Distributions / Analyzing Marginal Frequency Distributions / Building Marginal Relative Frequency Distributions / Analyzing Marginal Relative Frequency Distributions |  |

## 3 Modeling Linear Equations <br> Pacing: 19 Days

## Topic 1: Solving Linear Equations


 use given expressions to form equations with specific solution sets.


| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :--- | :--- | :--- | :--- | :--- |
| 3 |  | Tic-Tac-Bingo <br> Creating Linear <br> Equations | 8.EE.7.a <br> $8 . E E .7 . b$ | 1 | Students use their knowledge of <br> solving linear equations to create <br> linear equations with one solution, <br> no solution, or infinite solutions. <br> They play Tic-Tac-Bingo as they <br> work together to create equations <br> with given solution types from <br> assigned expressions. Then students <br> summarize the strategies they used <br> to create the equations. |

## Topic 2: Systems of Linear Equations



 solve additional systems, using the structure of the equations in the system to determine the most efficient solution strategy.

Standard: 8.EE. 8 Pacing: 12 Days

| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Crossing Paths <br> Point of Intersection of Linear Graphs | 8.EE.8.a | 2 | Students compare and analyze cost and income equations graphically and algebraically. They graph cost and income equations on the same graph to determine a point of intersection and interpret the point of intersection as the solution to two equations. Finally, students compare determining a point of intersection from a table alone with doing so using equations and a graph. | - The point of intersection is the point at which two lines cross on a coordinate plane. <br> - When one line represents the cost of an item and the other line represents the income from selling the item, the point of intersection is called the break-even point. <br> - The point of intersection of two linear graphs is the ordered pair that represents a solution to both equations of the graphs. |
| 2 | The Road Less Traveled <br> Systems of Linear Equations | 8.EE.8.b | 2 | Students write and analyze linear systems of equations. They informally calculate the solutions to systems of linear equations and then graph the systems of equations. Students conclude when parallel lines comprise the system, the lines will never intersect, so there is no solution to the system. They then determine whether the graphs of linear systems of equations are parallel, perpendicular, or neither through analysis of their slopes. | - A system of linear equations is formed when two or more linear equations define a relationship between quantities. <br> - The solution of a linear system is an ordered pair that is a solution to both equations in the system. <br> - Systems that have one or many solutions are called consistent systems. Systems with no solution are called inconsistent systems. <br> - The slopes of parallel lines are equal. |
| 3 | The County Fair <br> Using Substitution to Solve Linear Systems | 8.EE.8.a <br> 8.EE.8.b <br> 8.EE.8.C | 2 | Students learn to use the substitution method to solve systems of linear equations. They use substitution to solve systems of linear equations including systems with no solution or with infinite solutions. They define variables, write systems of equations, solve systems, and interpret the meaning of the solution in terms of the problem context. | - The substitution method is a process of solving a system of equations by substituting a variable in one equation with an equal expression. <br> - To use the substitution method, it is useful if at least one equation is written in slopeintercept form. <br> - Problem situations can be expressed using systems of equations and solved for unknown quantities using substitution methods. <br> - When a system has no solution, the equation resulting from the substitution step has no solution. <br> - When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions. |


| Lesson | Title I Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | Rockin' Roller Rinks <br> Choosing a Method to Solve a Linear System | 8.EE.8.a 8.EE.8.b 8.EE.8.c | 2 | Students compare the cost of holding a middle school skating event at three different locations. They write equations for each location and compare the cost for different numbers of skaters, by solving systems of equations, completing tables of values, and creating graphs. | - Problem situations are expressed using systems of equations and solved for unknown quantities using substitution methods. <br> - To use the substitution method, it is useful if at least one equation is written in slopeintercept form. <br> - When a system has no solution, the equation resulting from the substitution step has no solution and the graphed lines are parallel. <br> - When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions and the graphed lines are coincident. |
| Learnin MATHia | Individually with or Skills Practice | 8.EE.8.a 8.EE.8.b 8.EE.8.c | 4 | Students practice graphing and solving systems of linear equations. <br> MATHia Unit: Systems of Linear Equations <br> MATHia Workspaces: Introduction to Systems of Linear Equations / Modeling Linear Systems Involving Integers / Modeling Linear Systems Involving Decimals / Solving Linear Systems Using Substitution |  |

## 4 Expanding Number Systems

Pacing: 16 Days

## Topic 1: The Real Number System

Students build onto their knowledge of number systems to include the set of irrational numbers, specially square roots and cube roots of non-perfect squares and cubes. They review natural numbers, whole numbers, integers, and rational numbers and determine which systems have an additive identity, additive inverse, multiplicative identity, or multiplicative inverse and which are closed under the four basic arithmetic operations. They learn to convert rational numbers from repeating decimals to fractional form. Students then determine square roots and cube roots of perfect squares and perfect cubes and use rational approximations to estimate the value of irrational numbers, including solutions to equations.

| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | So Many Numbers, So Little Time <br> Number Sort | 8.NS. 1 | 1 | Students sort numbers and justify their reasoning. They analyze the work of their peers and provide reasoning for the way in which their peers grouped their numbers. | - Numbers can be grouped in a variety of ways according to their characteristics. <br> - Some ways that numbers can be identified are as whole numbers, natural numbers, integers, fractions, decimals, rational numbers, and irrational numbers. <br> - Some groups of numbers are subsets of larger groups of numbers. |
| 2 | Rational Decisions <br> Rational and Irrational Numbers | 8.NS. 1 | 2 | Students learn formal definitions for rational and irrational numbers. They conclude the set of rational numbers is closed and includes the set of whole numbers, the set of integers, some fractions, and some decimals. Then students write fractions as repeating decimals and convert terminating and repeating decimals to fractions. | - A number set is closed under an operation if combining any two members of the set using the given operation results in a member of the set. <br> - A number set can be closed under an operation but not closed under the inverse operation. <br> - Number sets are created to address lack of closure. <br> - A rational number is a number that can be written in the form $a b$, where $a$ and $b$ are both integers and $b$ is not equal to 0 . <br> - All rational numbers can be written as terminating or repeating decimals. <br> - A repeating decimal is a decimal that has one or more digits repeat indefinitely. <br> - A terminating decimal is a decimal that has a finite number of non-zero digits. <br> - A decimal that is not terminating non-repeating is an irrational number. <br> - Every terminating or repeating decimal can be converted to a rational number. |
| 3 | What Are Those?! <br> The Real Numbers | $\begin{aligned} & \text { 8.NS. } 2 \\ & \text { 8.EE. } 2 \end{aligned}$ | 2 | Students calculate square roots of perfect square and cube roots of perfect cubes. They locate irrational numbers on a number line between two rational numbers. Then they summarize the relationships among the sets of numbers in the real number system. | - Square roots of numbers that are not perfect squares and cube roots of numbers that are not perfect cubes are irrational numbers. <br> - The set of real numbers includes both the set of rational numbers and the set of irrational numbers. <br> - A Venn diagram can be used to represent the relationship between number sets. |
| Learnin MATHi | Individually with or Skills Practice | $\begin{aligned} & \text { 8.NS. } 1 \\ & \text { 8.NS. } 2 \\ & \text { 8.EE. } 2 \end{aligned}$ | 2 | Students determine perfect squares and their square root. They determine decimal approximations of square roots of nonperfect squares. Students classify real numbers as rational or irrational. Students practice comparing and ordering real numbers on a number line. <br> MATHia Unit: Rational and Irrational Numbers <br> MATHia Workspaces: Introduction to Irrational Numbers / Graphing Real Numbers on a Number Line / Ordering Rational and Irrational Numbers |  |

## Topic 2: Pythagorean Theorem


 Pythagorean Theorem to calculate distances between two points on the coordinate plane and in two- and three-dimensional geometric figures.

Standards: 8.EE.2, 8.G.6, 8.G.7, 8.G. 8 Pacing: 9 Days

| Lesson | Title I Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | The Right Triangle Connection <br> The Pythagorean Theorem | $\begin{aligned} & \text { 8.EE. } 2 \\ & \text { 8.G. } 6 \\ & \text { 8.G. } 7 \end{aligned}$ | 2 | Students learn about and prove the Pythagorean Theorem using area models. They practice using the Pythagorean Theorem to solve for unknown side lengths in mathematical and contextual problems. | - A right angle is an angle that measures $90^{\circ}$ and a right triangle is a triangle with exactly one right angle. <br> - The leg of a right triangle is one of the two shorter sides and the hypotenuse is the side opposite the right angle. <br> - The Pythagorean Theorem states that the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse. <br> - The Pythagorean Theorem states that if $a$ and $b$ are the lengths of the legs of a right triangle and $c$ is the length of the hypotenuse, then $a^{2}+b^{2}=c^{2}$. <br> - Area models can be used to prove the Pythagorean Theorem. <br> - The Pythagorean Theorem is used to determine unknown lengths in right triangles in mathematical and contextual problems. |
| 2 | Can That Be Right? <br> The Converse of the Pythagorean Theorem | $\begin{aligned} & \text { 8.EE. } 2 \\ & \text { 8.G. } 6 \\ & \text { 8.G. } 7 \end{aligned}$ | 2 | Students learn about and prove the Converse of the Pythagorean Theorem. They generate Pythagorean triples and use the Pythagorean Theorem and its converse to solve problems. | - The converse of a theorem is created when the if-then parts of that theorem are exchanged. <br> - The Converse of the Pythagorean Theorem states that if the sum of the squares of two sides of a triangle equals the square of the third side, then the triangle is a right triangle. <br> - A Pythagorean triple is any set of three positive integers that satisfy the equation $a^{2}+b^{2}=c^{2}$. <br> - Multiples of Pythagorean Triples are also Pythagorean Triples. <br> - The Pythagorean Theorem and its converse are used to solve mathematical and contextual problems. |
| 3 | Pythagoras Meets Descartes <br> Distances in a Coordinate System | 8.G. 8 | 1 | Students apply the Pythagorean Theorem to the coordinate plane. They calculate various distances using coordinates of points aligned either horizontally or vertically using subtraction and diagonal distances using the Pythagorean Theorem. | - The Pythagorean Theorem is used to determine the distance between two points on a coordinate plane. <br> - The Pythagorean Theorem states; if $a$ and $b$ are the lengths of the legs of a right triangle and $c$ is the length of the hypotenuse, then $a^{2}+b^{2}=c^{2}$. |


| Lesson Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: |
| 4Catty Corner <br> Side Lengths in Two <br> and Three Dimensions | $\begin{aligned} & \text { 8.EE. } 2 \\ & \text { 8.G. } 7 \end{aligned}$ | 2 | Students use the Pythagorean Theorem to determine the length of a three-dimensional diagonal of a rectangular solid. They also apply the Pythagorean Theorem to determine two-dimensional diagonals of rectangles and trapezoids. | - The Pythagorean Theorem is used to determine the length of diagonals in two- and three-dimensional figures. |
| Learning Individually with MATHia or Skills Practice | $\begin{aligned} & \text { 8.EE. } 2 \\ & \text { 8.G. } 6 \\ & \text { 8.G. } 7 \\ & \text { 8.G. } \end{aligned}$ | 2 | Student use the Pythagorean Theorem to solve real-world and mathematical problems. Students determine distances on the coordinate plane using the Pythagorean Theorem. <br> MATHia Unit: The Pythagorean Theorem <br> MATHia Workspaces: Exploring the Pythagorean Theorem / Applying the Pythagorean Theorem / Problem Solving Using the Pythagorean Theorem / Calculating Distances on the Coordinate Plane |  |

## 5 Applying Powers <br> Pacing: 23 Days

## Topic 1: Exponents and Scientific Notation

 powers, and develop rules for each operation. Then they determine rules for 0 and negative exponents.

 standard form.

Standards: 8.EE.1, 8.EE.3, 8.EE. 4 Pacing: 13 Days

| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | It's a Generational Thing <br> Properties of Powers with Integer Exponents | 8.EE. 1 | 3 | Students write and evaluate expressions with positive integer exponents. They begin with a context using the power with a base of 2 . Students then investigate positive and negative integer bases where the negative sign may or may not be raised to a power depending on the placement of parentheses. | - Large numbers that have factors that are repeated can be written as a product of powers. <br> - Placement of parentheses in an expression with an exponent determines what portion of the expression is raised to a power. <br> - When a negative value is raised to a power with an exponent that is an even integer, the simplified expression is a positive value. When a negative value is raised to a power with an exponent that is an odd integer, the simplified expression is a negative value. |
| 2 | Show What You Know <br> Analyzing Properties of Powers | 8.EE. 1 | 1 | Students organize and discuss properties of powers. They write mathematical justifications for each step within a solution path using the properties of powers. Students solve additional practice problems and examine student work for correctness. | - Large numbers that have factors that are repeated can be written as a product of powers. <br> - Placement of parentheses in an expression with an exponent determines what portion of the expression is raised to a power. <br> - When a negative value is raised to a power with an exponent that is an even integer, the simplified expression is a positive value. When a negative value is raised to a power with an exponent that is an odd integer, the simplified expression is a negative value. |
| 3 | The Big and Small of It <br> Scientific Notation | $\begin{aligned} & \text { 8.EE. } 3 \\ & \text { 8.EE. } 4 \end{aligned}$ | 2 | Students are introduced to scientific notation. They convert from standard form to scientific notation and from scientific notation to standard form and begin comparing numbers written in scientific notation. | - Scientific notation is a mathematical notation used to write and compare very large and very small numbers. <br> - Scientific notation is a way to express a very large or very small number as the product of a number greater than or equal to 1 and less than 10 and a power of 10. <br> - Scientific notation allows for easy recognition of the magnitude of values for comparison purposes. |


| Lesson Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: |
| How Much Larger? <br> Operations with Scientific Notation | $\begin{aligned} & \text { 8.EE. } 3 \\ & \text { 8.EE. } 4 \end{aligned}$ | 2 | Students perform operations on numbers represented in scientific notation with and without context. They connect the Product Rule of Powers and the Quotient of a Power Rule with scientific notation. Students perform operations with numbers written in scientific notation. They also compare and operate on numbers expressed in standard form or scientific notation. | - Mathematical operations can be performed efficiently on numbers written in scientific notation. <br> - The Product Rule of Powers and the Quotient of a Power Rule can be applied to numbers written in scientific notation. <br> - When multiplying or dividing numbers written in scientific notation, multiply or divide the mantissas and apply the appropriate property of exponents to the characteristics. Rewrite the result in scientific notation if needed. <br> - When adding or subtracting numbers written in scientific notation, write both numbers so that they have the same characteristic. Add or subtract the mantissas and keep the characteristic. Rewrite the result in scientific notation if needed. |
| Learning Individually with MATHia or Skills Practice | 8.EE. 1 <br> 8.EE. 3 <br> 8.EE. 4 | 5 | Students practice simplifying mathematical expressions using the rules of exponents. Students convert between and compare numbers written in standard form and scientific notation. <br> MATHia Unit: Properties of Whole Number Exponents <br> MATHia Workspaces: Introduction to the Power Rules / Using the Product Rule and the Quotient Rule / Using the Power to a Power Rule / Using the Product to a Power Rule and the Quotient to a Power Rule / Using Properties of Exponents with Whole Number Powers / Rewriting Expressions with Negative and Zero Exponents <br> MATHia Unit: Scientific Notation <br> MATHia Workspaces: Using Scientific Notation / Comparing Numbers Using Scientific Notation |  |

## Topic 2: Volume of Curved Figures

Students derive formulas for the volumes of cylinders, cones, and spheres. They apply each formula to mathematical and real-world problems. In some cases, students must use the
 cones and cylinders as they solve problems.

Standard: 8.G. 9 Pacing: 10 Days

| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Drum Roll, Please! <br> Volume of a Cylinder | 8.G.9 | 2 | Students explore how to determine the volume of a cylinder. They identify characteristics of a cylinder such as the radius, diameter, and height. Using circular discs, they calculate the volume of the cylinder. Students then answer several questions related to the formula used to compute the volume of right and oblique cylinders. | - A cylinder is a three-dimensional object with two parallel, congruent, circular bases. <br> - A right circular cylinder is a cylinder in which the bases are circles and are aligned one directly above the other. <br> - The formula for the volume of the cylinder is written two different ways: $V=B h$, where $V$ is the volume of the cylinder, $B$ is the area of the base of the cylinder, and $h$ is the height of the cylinder and $V=\pi\left(r^{2}\right) h$, where $V$ is the volume of the cylinder, $r$ is the length of the radius of the base of the cylinder, and $h$ is the height of the cylinder. |
| 2 | Cone of Silence <br> Volume of a Cone | 8.G.9 | 2 | Students learn how to calculate the volume of a cone and how to solve problems involving cones. They use the formulas for the volume of a cylinder and the volume of a pyramid to write formulas for the volume of a cone. They solve problems when given different dimensions of a cone, including the slant height. | - A cone is a three-dimensional object with a circular base and one vertex. <br> - The formula for the volume of the cone can be written two different ways: $V=(1 / 3) B h$, where $V$ is the volume of the cone, $B$ is the area of the base of the cone, and h is the height of the cone and $V=(1 / 3) \pi\left(r^{2}\right) h$, where $V$ is the volume of the cone, $r$ is the length of the radius of the base of the cone, and $h$ is the height of the cone. <br> - The Pythagorean Theorem is used to determine the height of a cone when given the radius and slant height of the cone. |
| 3 | Pulled in All Directions <br> Volume of a Sphere | 8.G.9 | 2 | Students investigate the volume of a sphere. They derive and then use the formula for the volume of a sphere to calculate the volume of various spheres. | - A sphere is defined as the set of all points in three dimensions that are equidistant from a given point called the center. <br> - The formula for the volume of a sphere is $V=4 / 3(\pi) r^{3}$, where $V$ is the volume of the sphere and $r$ is the length of the radius of the sphere. |


| Lesson | Title / Subtitle | Standards | Pacing* | Lesson Summary | Essential Ideas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | Silos, Frozen <br> Yogurt, and <br> Popcorn <br> Volume Problems with Cylinders, Cones, and Spheres | 8.G. 9 | 2 | Students review all of the formulas they have learned up to this point and use them to solve real-world and mathematical problems. Students determine the volume of grain needed to fill a silo, the volume of a cone and a melted scoop of yogurt, and the volume of cylindrical and rectangular-prism shaped popcorn containers. | - The formula for the volume of a cylinder is $V=(\pi)\left(r^{2}\right) h$, where $V$ is the volume of the cylinder, $r$ is the length of the radius of the base of the cylinder, and $h$ is the height of the cylinder. <br> - The formula for the volume of a cone is $V=1 / 3(\pi)\left(r^{2}\right) h$, where $V$ is the volume of the cone, $r$ is the length of the radius of the base of the cone, and $h$ is the height of the cone. <br> - The formula for the volume of a sphere is $V=4 / 3(\pi) r^{3}$, where $V$ is the volume of the sphere and $r$ is the length of the radius of the sphere. |
| Learnin MATHia | Individually with or Skills Practice | 8.G. 9 | 2 | Students apply the formulas for the volume of a cylinder, cone, and sphere to solve a variety of problems. <br> MATHia Unit: Volume <br> MATHia Workspaces: Relating Volumes of Cylinders, Cones, and Spheres / Calculating Volume of Cylinders / Using Volume of Cylinders / Calculating Volume of Cones / Using Volume of Cones / Calculating Volume of Spheres / Using Volume of Spheres |  |

## Total Days: 137

## Learning Together: 101

## Learning Individually: 36

