1 Thinking Proportionally

Pacing: 39 Days

Topic 1: Circles and Ratio

Students learn formulas for the circumference and area of circles and use those formulas to solve mathematical and real-world problems. Students also learn that the irrational number pi (π) is the ratio of a circle's circumference to its diameter.

Standard: 7.G.4 **Pacing:** 7 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Pi: The Ultimate Ratio** Exploring the Ratio of Circle Circumference to Diameter	7.G.4	2	Students explore the relationship between the distance around and the distance across various circles. They notice that for every circle the ratio of the circumference to diameter is pi.	• The circumference of a circle is the distance around the circle. • The ratio of the circumference of a circle to the diameter of a circle is approximately 3.14 or pi. • The formula for calculating the circumference of a circle is $C = d\pi$ or $C = 2\pi r$ where C is the circumference of a circle, d is the length of the diameter of the circle, r is the length of the radius of the circle, and π is represented using the approximation 3.14.
2	**That's a Spicy Pizza!** Area of Circles	7.G.4	1	Students explore the area of a circle in terms of its circumference. They derive the area for a circle and then solve problems using the formulas for the circumference and area for circles	• If a circle is divided into equal parts, separated, and rearranged to resemble a parallelogram, the area of a circle can be approximated by using the formula for the area of a parallelogram with a base length equal to half the circumference and a height equal to the radius. • The formula for calculating the area of a circle is $A = \pi r^2$ where A is the area of a circle, r is the length of the radius of the circle, and π is represented using the approximation 3.14. • When solving problems involving circles, remember that the circumference formula is used to determine the distance around a circle, while the area formula is used to determine the amount of space contained inside a circle.
3	**Circular Reasoning** Solving Area and Circumference Problems	7.G.4	2	Students use the area of a circle formula and the circumference formula to solve for unknown measurements in real-world and mathematical problem.	• The formula to calculate the area of a circle is $A = \pi r^2$. • The formula to calculate the circumference of a circle is $C = 2\pi r$. • Composite figures that include circles are used to solve for unknowns.
Learning Individually with MATHia or Skills Practice		7.G.4	2	Students practice solving problems involving area and circumference of circles. **MATHia Unit:** Circles **MATHia Workspaces:** Investigating Circles / Calculating Circumference and Area of Circles	

*Pacing listed in 45-minute days

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Topic 2: Fractional Rates

Students calculate and use unit rates from ratios of fractions. They review strategies for solving proportions and then use means and extremes to solve real-world proportion problems.

Standards: 7.RP.1, 7.RP.2.c, 7.RP.3 **Pacing:** 6 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Making Punch** Unit Rate Representations	7.RP.1	1	Students recall the concepts of ratio and unit rate and how to represent these mathematical objects using tables and graphs. Students use the unit rate as a measure of a qualitative characteristic: the strength of the lemon-lime taste of a punch recipe. They represent this measure in tables and graphs and with fractions in the numerator.	• A rate is a ratio that compares two quantities that are measured in different units. • A unit rate is a comparison of two measurements in which the denominator has a value of one unit. • Tables are used to represent equivalent ratios. • Graphs can be used to represent rates.
2	**Eggzactly!** Solving Problems with Ratios of Fractions	7.RP.1	1	Students determine ratios and write rates, including complex ratios and rates. Students will write proportions and use rates to determine miles per hour. They will scale up and scale down to determine unknown quantities.	• A complex ratio has a fractional numerator or denominator (or both). • Complex ratios and rates can be used to solve problems.
3	**Tagging Sharks** Solving Proportions Using Means and Extremes	7.RP.2.c 7.RP.3	2	Students solve several proportions embedded in real world contexts. Several proportions that contain one variable are solved using one of three methods: the scaling method, the unit rate method, and the means and extremes method. Students learn to isolate a variable in an equation by using inverse operations.	• A variable is a letter or symbol used to represent a number. • To solve a proportion means to determine all the values of the variable that make the proportion true. • A method for solving a proportion called the scaling method involves multiplying (scaling up) or dividing (scaling down) the numerator and denominator of one ratio by the same factor until the denominators of both ratios are the same number. • A method for solving a proportion called the unit rate method involves changing one ratio to a unit rate and then scaling up to the rate you need. • A method for solving a proportion called the means and extremes method involves identifying the means and extremes, and then setting the product of the means equal to the product of the extremes to solve for the unknown quantity. • Isolating a variable involves performing an operation, or operations, to get the variable by itself on one side of the equals sign. • Inverse operations are operations that undo each other such as multiplication and division, or addition and subtraction.
Learning Individually with MATHia or Skills Practice		7.RP.1 7.RP.2.c	2	Students determine and compare unit rates. They solve proportions using equivalent ratios and means and extremes. **MATHia Unit:** Ratio and Rate Reasoning **MATHia Workspaces:** Fractional Rates / Determining and Comparing Unit Rates **MATHia Unit:** Proportional Reasoning **MATHia Workspaces:** Proportional Relationships / Determining Characteristics of Graphs and Proportional Relationships / Solving Proportions Using Equivalent Ratios / Rewriting Proportions as Products / Solving Proportions Using Means and Extremes	

*Pacing listed in 45-minute days

Topic 3: Proportionality

Students differentiate between proportional and non-proportional relationships, including linear relationships that are not proportional. They identify and use the constant of proportionality from tables, graphs, equations, and real-world situations; represent proportional relationships with equations; and explain the meaning of points on the graph of a proportional relationship.

Standard: 7.RP.A.2 **Pacing:** 11 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**How Does Your Garden Grow?** Proportional Relationships	7.RP.2.a	2	Students explore graphs and tables of proportional and non-proportional relationships. They determine that the graphs of proportional relationships are straight lines that pass through the origin. They also learn that tables of proportional relationships have a constant ratio of corresponding values of the quantities. Students learn the term direct variation and relate it to proportional relationships.	• Graphs of equivalent ratios for a straight line that passes through the origin. • Linear relationships are also proportional relationships if the ratio between corresponding values of the quantities is constant. • The graph of a proportional relationship is a straight line that passes through the origin. • A linear relationship represents a direct variation if the ratio between the output values and input values is a constant. The quantities are said to vary directly. • Multiple representations such as tables and graphs are used to show examples of proportional, or direct variation, relationships between two values within the context of real-world problems.
2	**Complying with Title IX** Constant of Proportionality	7.RP.2.b 7.RP.2.c	2	Students explore equations of proportional relationships. They determine the constant of proportionality, the constant ratio of the outputs to the inputs in a proportional relationship. Students explore the reciprocal relationship of constants of proportionality in equations. They use the constant of proportionality to write and solve equations.	• In a proportional relationship, the ratio between two quantities is always the same. It is called the constant of proportionality. • The constant of proportionality in a proportional relationship is the ratio of the outputs to the inputs. • In a proportional relationship, two different proportional equations can be written. The coefficients, or constants of proportionality, in the two equations are reciprocals. • The equation used to represent the proportional relationship between two values is $y = kx$, where x and y are the quantities that vary, and k is the constant of proportionality. • Proportional relationships are used to write equations and solve for unknown values.
3	**Fish-Inches** Identifying the Constant of Proportionality in Graphs	7.RP.2.b 7.RP.2.d	1	Students analyze real world and mathematical situations, both proportional and non-proportional, represented on graphs and then identify the constant of proportionality when appropriate. Throughout the lesson, students interpret the meaning of points on graphs in terms of a proportional relationship, including the meaning of (1, y) and (0, 0).	• The graph of two variables that are proportional, or that vary directly, is a line that passes through the origin, (0, 0). • The ratio of the y-coordinate to the x-coordinate (their quotient) for any point is equivalent to the constant of proportionality, k, when analyzing a graph of two variables that are proportional. • When analyzing the graph of two variables that are not proportional, the ratios of the y-coordinate to the x-coordinate for any points are not equivalent.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Minding Your Ps and Qs** Constant of Proportionality in Multiple Representations	7.RP.2	2	Students use proportional relationships to create equivalent multiple representations, such as diagrams, equations, tables, and graphs of the situation. A proportional relationship may initially be expressed using only words, or a table of values, or an equation, or a graph.	• The graph of two variables that are proportional, or that vary directly, is a line that passes through the origin, (0, 0). • When analyzing the table of two variables that vary directly, the ratios of the y-value to the x-value for any pair are equivalent. • The equation used to represent a proportional relationship between two values is $y - kx$, where x varies directly as y, and k is the constant of proportionality. • A table of equivalent ratios, a graph of a straight line through the origin, and an equation of the form $y - kx$ can be created to represent a scenario describing quantities in a proportional relationship.
Learning Individually with MATHia or Skills Practice		7.RP.2.a 7.RP.2.b 7.RP.2.c	4	Students write ratios and determine the constant of proportionality in real-world problems. They practice determining the constant of proportionality, writing equations, and drawing a line to represent the direct variation equation to solve problems. Students are given graphs to determine if it represents a direct variation. **MATHia Unit:** Representing Proportional Relationships by Equations **MATHia Workspaces:** Exploring Proportions / Writing Proportional Relationships with Equations / Converting Between Forms of Proportional Relationships / Modeling the Constant of Proportionality	

*Pacing listed in 45-minute days

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Topic 4: Proportional Relationships

Students use proportions and percent equations to solve real-world problems about money and scale drawings. They use multiple representations to solve and compare percents. Then students use proportionality to solve problems with scale drawings and scale factors.

Standards: 7.RP.3, 7.G.1 **Pacing:** 15 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Markups and Markdowns** Introducing Proportions to Solve Percent Problems	7.RP.3	2	Students analyze strategies for determining the unknown value in a percent problem. Students use proportions to solve percent problems. They connect percent problems with direct variation and proportional relationships.	• Tape diagrams are used to solve percent problems. • Proportions are used to solve percent problems. • Part-to-whole ratios are used to solve percent problems. • Proportions can be used to solve markdown and markup problems. • Multiple strategies can be used to solve percent problems with proportions. • Percent problems are related to direct variation within the context of real-world situations. • Proportional relationships can be represented by equations.
2	**Perks of Work** Calculating Tips, Commission, and Simple Interest	7.RP.3	2	Students solve proportions and percent equations in the context of tipping and commissions. They analyze both strategies as they determine the amount of a tip or commission, the percent tip or commission, and the total sale when given the percent and the tip or commission amount.	• Proportions are used to solve percent problems. • A proportion used to solve a percent problem is often written in the form percent = part / whole. • Percent equations are used to solve percent problems. • A percent equation can be written in the form percent x whole = part • Percent problems are related to direct variation within the context of real world situations. • Proportional relationships can be represented by an equation, a table, or a graph.
3	**No Taxation Without Calculation** Sales Tax, Income Tax, and Fees	7.RP.3	2	Students use percents to solve sales tax, income tax, and fee problems. They identify the percent relationship between two amounts as a proportional relationship, with a unit rate and constant of proportionality.	• Proportional relationships are the basis for solving percent problems in a real-world context. • Sales tax is a percentage of the selling prices of many goods or services that is added to the price of an item. The percentage of sales tax varies by state, but it is generally between 4% and 7%. • Income tax is a percentage of a person's or company's earnings that is collected by the state and national government.
4	**More Ups and Downs** Percent Increase and Percent Decrease	7.RP.3 7.G.6	2	Students compute percent increase and percent decrease in several situations. They apply percent increase and decrease to solving problems involving geometric measurement.	• Percent increase occurs when the new amount is greater than the original amount. To computer the percent increase, divide the amount of increase by the original amount. • Percent decrease occurs when the new amount is less than the original amount. To compute the percent increase, divide the amount of decrease by the original amount.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
5	**Pound for Pound, Inch for Inch** Scale and Scale Drawings	7.G.1	3	Students use scale models to calculate measurements and enlarge and reduce the size of models. They enlarge or reduce the size of objects and calculate relevant measurements, explore scale drawings, and describe the meaning of several different scales. Students then determine which scale will produce the largest and smallest drawing of an object when different units of measure are given.	• Scale drawings are representations of real objects or places that are in proportion to the real objects or places they represent. • The scale of a drawing is the ratio drawing length : actual length. • The scale of a map is the ratio map distance : actual distance. • When calculating the area of a scaled figure, the scale must be applied to all dimensions of the figure.
Learning Individually with MATHia or Skills Practice		7.RP.3 7.G.1	4	Students practice converting between fractions, decimals, and percents. They solve percent problems for the part, the percent, or the whole, and solve percent change problems. Students use scale factors to determine unknown measures given real-life situations. **MATHia Unit:** Percent Conversions **MATHia Workspaces:** Fractional Percent Models / Converting with Fractional Percents **MATHia Unit:** Proportional Reasoning and Percents **MATHia Workspaces:** Using Proportions to Solve Percent Problems / Solving Simple Percent Problems **MATHia Unit:** Problem Solving with Percents Using Proportional Relationships **MATHia Workspaces:** Calculating Percent Change and Final Amounts / Using Percents and Percent Change **MATHia Unit:** Calculating Sales Tax and Discounts **MATHia Workspaces:** Calculating Sales Tax or Discounts / Solving Problems with Both Sales Tax and Discounts / Analyzing Different Forms of Expressions **MATHia Unit:** Scale Drawings **MATHia Workspaces:** Critical Attributes of Similar Figures / Using Scale Drawings / Using Scale Factor	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

2 Operating with Signed Numbers

Pacing: 17 Days

Topic 1: Adding and Subtracting Rational Numbers

Students use physical motion, number lines, and two-color counters to develop conceptual understanding of adding and subtracting integers. They develop rules for these operations and apply the rules to the set of rational numbers.

Standards: 7.NS.1, 7.NS.3 **Pacing:** 9 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Math Football** Using Models to Understand Integer Addition	7.NS.1	1	A math football game is used to model the sum of a positive and negative integer. Students use number cubes to generate the integers. They will then use that information and write integer number sentences.	• A model can be used to represent the sum of a positive and negative integer, two negative integers, or two positive integers. • Information from a model can be written as an equation.
2	**Walk the Line** Adding Integers, Part I	7.NS.1.b	2	Students explore patterns for adding two integers using a number line. They focus on the absolute values of the numbers being added and develop informal rules for adding integers.	• On a number line when adding a positive integer, move to the right. • On a number line, when adding a negative integer, move to the left. • When adding two positive integers, the sign of the sum is always positive. • When adding two negative integers, the sign of the sum is always negative. • When adding a positive and a negative integer, the sign of the sum is the sign of the number that is the greatest distance from zero on the number line.
3	**Two-Color Counters** Adding Integers, Part II	7.NS.1.a 7.NS.1.b	2	Students use two-color counters to develop rules for adding integers. They model adding positive and negative integers with the two-color counters. Students use a graphic organizer to represent how to add additive inverses using a variety of representations.	• Opposite quantities in real-life situations combine to make 0. Examples include temperature change, water level, weight change, and floors above and below ground floor. • Two numbers with the sum of zero are called additive inverses. • Addition of integers is modeled using two-color counters that represent positive charges (yellow counters) and negative charges (red counters). • When two integers have the same sign and are added together, the sign of the sum is the sign of both integers. • When two integers have the opposite sign and are added together, the integers are subtracted and the sign of the sum is the sign of the integer with the greater absolute value.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**What's the Difference?** Subtracting Integers	7.NS.1.c	2	Students use number lines and two-color counters to model subtraction of signed numbers. They develop and apply rules for subtracting integers.	• Subtraction can mean to take away objects form a set. Subtraction also describes the difference between two numbers. • A zero pair is a pair of two-color counters composed of one positive counter (+) and one negative counter (–). • Adding zero pairs to a two-color counter representation of an integer does not change the value of the integer. • Subtraction of integers is modeled using two-color counters that represent positive charges (yellow counters) and negative charges (red counters). • Subtraction of integers is modeled using a number line. • Subtracting two negative integers is similar to adding two integers with opposite signs. • Subtracting a positive integer from a positive integer is similar to adding two integers with opposite signs. • Subtracting a positive integer from a negative integer is similar to adding two negative integers. • Subtracting two integers is the same as adding the opposite of the subtrahend, number you are subtracting.
5	**All Mixed Up** Adding and Subtracting Rational Numbers	7.NS.3	1	Students apply their knowledge of adding and subtracting positive and negative integers to the set of rational numbers.	• The rules for operating on integers also apply to operating on rational numbers.
Learning Individually with MATHia or Skills Practice		7.NS.1	1	Students practice adding and subtracting integers using a number line. **MATHia Unit:** Integer Operations **MATHia Workspaces:** Understanding Opposites / Adding and Subtracting Negative Integers / Using Number Lines to Add and Subtract Integers / Developing Algorithms for Adding and Subtracting Integers	

*Pacing listed in 45-minute days

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Topic 2: Multiplying and Dividing Rational Numbers

Students use number lines and two-color counters to model and develop rules for the signs of the products and quotients of signed numbers. They convert rational numbers from fractional to decimal form. Then students apply rules and properties to the set of rational numbers.

Standards: 7.NS.1.d, 7.NS.2, 7.NS.3, 7.RP.3 **Pacing:** 8 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Equal Groups** Multiplying and Dividing Integers	7.NS.2.a 7.NS.3	2	Students use number lines and two-color counters to model the product of two integers. They use the models to develop rules for multiplying integers. Using fact families, students apply rules for the multiplication of integers to the division of integers.	• Multiplication can be thought of as repeated addition. • Multiplication of integers can be modeled using two-color counters that represent positive charges (yellow counters) and negative charges (red counters). • Multiplication of integers can be modeled using a number line. • The product that results from multiplying two positive integers is always positive. • The product that results from multiplying two negative integers is always positive. • The product that results from multiplying a negative integers and a positive is always negative. • The product that results from multiplying an odd number of negative integers is always negative. • The product that results from multiplying an even number of negative integers is always positive. • Division and multiplication are inverse operations. • The algorithms for determining the sign of the quotient when performing division are the same as the algorithms for determining the sign of the product when performing multiplication.
2	**Be Rational!** Quotients of Integers	7.NS.2.b 7.NS.2.d	1	Students write the quotients of integers as fractions and decimals. They learn that the decimal representation of rational numbers terminates or repeats.	• Decimals are classified as terminating and non-terminating. Non-terminating decimals are classified as repeating or non-repeating. • Bar notation is used when writing repeating decimals. • The quotient of two integers, when the divisor is not zero, is a rational number. • The sign of a negative rational number in fractional form can be placed in front of the fraction, in the numerator of the fraction or in the denominator of the fraction.
3	**Building a Wright Brothers' Flyer** Simplifying Expressions to Solve Problems	7.NS.3 7.RP.3	2	Students solve real-world problems involving numeric expressions and signed rational numbers, including problems about percent error.	• Expressions and equations composed of rational numbers are used to solve real-world problems.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Properties Schmoperties** Using Number Properties to Interpret Expressions with Signed Numbers	7.NS.1.d 7.NS.2.c 7.NS.3	1	Students apply the distributive property to expanding and factoring with -1 and learn that subtraction is the same as adding the opposite. They identify properties in expressions with rational coefficients.	• Number properties are used to solve mathematical problems. • The opposite of an expression can be modeled as a reflection across 0 on the number line. • The opposite of an expression is the same as the expression with –1 factored out. • Number properties can be used to operate with rational numbers in order to make the computations more efficient. • Subtraction of an integer can be written as the addition of the opposite of that integer.
Learning Individually with MATHia or Skills Practice		7.NS.2 7.NS.3	2	Students use fact families to explore dividing integers. They then practice simplifying a variety of numeric expressions using order of operations. **MATHia Unit:** Integer Operations **MATHia Workspaces:** Multiplying and Dividing Integers / Converting Rational Numbers to Decimals **MATHia Unit:** Evaluating Numeric Expressions **MATHia Workspaces:** Evaluating Simple Numeric Expressions with Integers / Evaluating Numeric Expressions Involving Integers with Parentheses and Exponents / Evaluating Simple Numeric Expressions with Rational Numbers / Evaluating Complex Numeric Expressions with Rational Numbers	

*Pacing listed in 45-minute days

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3 Reasoning Algebraically

Pacing: 38 Days

Topic 1: Algebraic Expressions

Students explore algebraic expressions with rational coefficients. They apply the Distributive Property as a strategy to write equivalent expressions and to factor linear expressions. Students combine like terms, including like linear terms, and use properties of operations to add and subtract expressions.

Standards: 7.EE.1, 7.EE.2, 7.EE.3 **Pacing:** 7 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**No Substitution for Hard Work** Evaluating Algebraic Expressions	7.EE.3	1	Students review variables, algebraic expressions, and evaluating algebraic expressions. They practice evaluating expressions with rational numbers.	• A variable is a letter or symbol that is used to represent an unknown quantity. • An algebraic expression is a mathematical phrase involving at least one variable, and it can contain numbers and operational symbols. • A linear expression, with respect to the variable *x*, is a sum of terms which are rational numbers or rational numbers times *x*. • To evaluate an expression, replace each variable in the expression with numbers and then perform all possible mathematical operations.
2	**Mathematics Gymnastics** Rewriting Expressions Using the Distributive Property	7.EE.1	2	Students rewrite algebraic expressions with rational coefficients using the Distributive Property. They then expand linear expressions. Students will factor linear expressions in a variety of ways, including by factoring out the greatest common factor and the coefficient of the variable.	• The Distributive Property provides ways to write numerical and algebraic expressions in equivalent forms. • The Distributive Property states that if *a*, *b*, and *c* are any real numbers, then $a(b + c) = ab + ac$. • The Distributive Property is used to expand expressions. • The Distributive Property is used to factor expressions. • To factor an expression means to rewrite the expression as a product of factors. • A coefficient is the number that is multiplied by a variable in an algebraic expression. • A common factor is a number or an algebraic expression that is a factor of two or more numbers or algebraic expressions. • The greatest common factor is the largest factor that two or more numbers or terms have in common. • An expression can be factored in an infinite number of ways.
3	**All My Xs** Combining Like Terms	7.EE.1 7.EE.2	2	Students simplify expressions by combining like terms, with integer, fraction, and decimal coefficients. They use properties to simplify the expressions. Students add and subtract algebraic expressions, using addition of the opposite to subtract.	• A coefficient is the number that is multiplied by a variable in an algebraic expression. • Terms are considered like terms if their variable portions are the same. Like terms can be combined.
Learning Individually with MATHia or Skills Practice		7.EE.1	2	Students model the product of two factors and explore different factors of expressions. They then use the Distributive Property to factor and expand expressions. Students simplify variable expressions by combining like terms, by using number properties, and by using order of operations. **MATHia Unit:** Variable Expressions **MATHia Workspaces:** Factoring Linear Expressions / Rewriting Simple Algebraic Expressions Involving Integer Coefficients / Rewriting Algebraic Expressions Involving Integer Coefficients with Four Operations / Rewriting Algebraic Expressions Involving Integer Coefficients with Parentheses and Exponents / Rewriting Complex Algebraic Expressions Involving Integer Coefficients / Rewriting Algebraic Expressions Involving Integer Coefficients	

*Pacing listed in 45-minute days

Topic 2: Two-Step Equations and Inequalities

Students use bar models and double number lines to reason about and solve two-step linear equations. Then they use inverse operations to fluently and efficiently solve two-step equations with rational coefficients. Finally, students investigate, solve, and graph two-step inequalities.

Standard: 7.EE.4 **Pacing:** 16 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Picture Algebra** Modeling Equations by Equal Expressions	7.EE.4.a	2	Students use bar models to write expressions and equations and to solve for unknown quantities.	• An equation is a statement created by placing an equals sign between two expressions. • Algebraic expressions and equations represent relationships between values. • Equations can be modeled using bar models. • To solve an equation with a variable is to determine a value for the variable that makes the statement true.
2	**Expressions That Play Together ...** Solving Equations on a Double Number Line	7.EE.4.a	1	Students model contextual and mathematical situations using double number lines.	• An equation is a statement created by placing an equals sign between two expressions. • Algebraic expressions and equations represent relationships between values. • Equations can be modeled using double number lines. • To solve an equation with a variable is to determine a value for the variable that makes the statement true.
3	**Formally Yours** Using Inverse Operations to Solve Equations	7.EE.4.a	3	Students formalize the process and language of solving two-step equations. They refer to the Properties of Equality and use inverse operations flexibly to solve equations. Students consider efficient strategies for solving problems with different types of rational numbers. They practice solving two-step equations.	• A solution to an equation is any variable value that makes the equation true. • The Properties of Equality state that if an operation is performed on both sides of the equation, to all terms of the equation, the equation maintains its equality. • When the Properties of Equality are applied to an equation, the transformed equation will have the same solution as the original equation. • Inverse operations are pairs of operations that reverse each other such as addition and subtraction or multiplication and division. • Two-step equations are equations that require only two operations to solve them, addition or subtraction and multiplication or division. • In order to solve a two-step equation, the variable is isolated by applying inverse operation. • Strategies to improve equation-solving efficiency include terms of an equation with fractions by the least common denominator, multiplying the terms of an equation with decimals by the appropriate multiple of 10, and dividing out a common factor of the terms of an equation. • To determine if a solution to an equation is correct, substitute the value of the variable back into the original equation and if the equation remains equivalent, the solution is correct.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Be Greater Than** Solving Inequalities with Inverse Operations	7.EE.4.b	3	Students solve inequalities. They compare solving equations with solving inequalities. Students informally develop the Properties of Inequalities before analyzing them formally. They then solve and graph inequalities.	• An inequality is any mathematical sentence that has an inequality symbol such as >, <, ≥ or ≤ . • The graph of an inequality in one variable is the set of all points on a number line that make the inequality true. • The solution set of an inequality is the set of all points that make the inequality true. • An open circle on the number line graph of an inequality indicates the value circled is not part of the solution set, whereas a closed circle indicates the value circled is part of the solution set. • To solve an inequality means to determine the values of the variable that make the inequality true. • To solve an inequality, isolate the variable using the same algebraic steps that are used to solve an equation. • The Properties of Inequalities explain how an inequality relationship is maintained or changed when the same operation is performed on both sides of the inequality. • The inequality symbol remains the same when adding, subtracting, and multiplying or dividing an inequality by a positive number. • The inequality symbol reverses when multiplying or dividing an inequality by a negative number.
Learning Individually with MATHia or Skills Practice		7.EE.4	7	Students create visual models to represent real-world situations and solve using reasoning. They then write and solve expressions from a given real-world problem using tables. Students solve a variety of two-step equations and inequalities using formal strategies. **MATHia Unit:** Modeling Two-Step Expressions and Equations **MATHia Workspaces:** Using Picture Algebra with Equations / Identifying Attributes of Linear Relationships / Analyzing Models of Two-Step Linear Relationships / Modeling Two-Step Expressions **MATHia Unit:** Solving Two-Step Equations **MATHia Workspaces:** Checking Solutions to Linear Equations / Solving with Multiplication (No Type In) / Solving with Multiplication (Type In) / Solving with Division (No Type In) / Solving with Division (Type In) / Solving Two-Step Equations **MATHia Unit:** Solving Two-Step Inequalities **MATHia Workspaces:** Graphing Inequalities with Rational Numbers / Solving One-Step Linear Inequalities / Solving Two-Step Linear Inequalities **MATHia Unit:** Problem Solving with Two-Step Equations and Inequalities **MATHia Workspaces:** Determining the Value of an Independent Variable / Using Linear Equations and Inequalities / Solving Problems with Integers / Solving Problems with Decimals and Fractions / Graphs of Equations / Using Graphs to Solve Equations	

*Pacing listed in 45-minute days

Topic 3: Multiple Representations of Equations

Students use tables, graphs, verbal descriptions, and scenarios to write and analyze two-step linear equations and inequalities. They make connections across representations, interpreting graphs and equations in terms of the problem situation.

Standards: 7.EE.2, 7.EE.4 **Pacing:** 15 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Put It on the Plane** Representing Equations with Tables and Graphs	7.EE.4.a	2	Students analyze linear equations using tables and graphs. They write and solve equations, create tables of values, and create graphs of the situations. They use the graphs to answer questions about the situations. Students explain if the linear situations represent proportional relationships.	• A real-world linear problem situation can be expressed using multiple representations. • A real-world linear problem situation can be represented as a sentence, as a table, as a graph, and as an equation. • An equation provides information about the graph of the problem situation. • Negative numbers are used to represent time that has already elapsed, or the past tense.
2	**Stretches, Stacks, and Structure** Structure of Linear Equations	7.EE.2 7.EE.4.a	3	Students write and solve equations for more complicated contexts. They use tables to create equations that require the use of the expression (n – 1) to represent the quantity of the independent variable except for the initial value. They compare the two forms of the same equation and relate the equations to the graphs.	• More complex equations may require use of the distributive property and/or combining like terms in order to simplify an equation to a two-step equation. Then, practiced methods for solving the two-step equation can be used to complete the problem. • When writing an equation to represent a table of values, it is sometimes the case that the table of values grows at a constant rate, but the initial value is different than the constant rate of growth. When that is the case, the expression (n – 1) is used to represent the quantity of the independent variable except for the initial value. • Writing a linear equation in a different form can reveal information about the problem situation. • Writing a linear equation in the form $y = ax + b$ reveals the y-intercept of the graph of the problem situation.
3	**Deep Flight I** Building Inequalities and Equations to Solve Problems	7.EE.4	2	Students work with a negative rate of change. They use negative values to create a table and graph. Students then write and analyze an equations and inequalities with negative unit rates of change.	• The unit rate of change is the amount that the dependent value changes for every one unit that the independent value changes. • Multiple representations such as a table, an equation, and a graph are used to represent a problem situation.

*Pacing listed in 45-minute days

*Pacing listed in 45-minute days

CARNEGIE LEARNING

4 Analyzing Populations and Probabilities

Pacing: 25 Days

Topic 1: Introduction to Probability

Students conduct probability experiments with familiar objects and determine theoretical and experimental probabilities of simple events. They learn about using uniform and non-uniform probability models to organize the probabilities of the outcomes in a sample space. Students use proportional reasoning to predict expected frequencies of favorable outcomes in larger samples and to calculate the percent error between theoretical and experimental probabilities. They also use simulation with a variety of tools to simulate the results of experiments.

Standards: 7.SP.5, 7.SP.6, 7.SP.7, 7.RP.3 **Pacing:** 9 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Rolling, Rolling, Rolling ...** Defining and Representing Probability	7.SP.5	2	Students conduct an experiment that involves rolling one six-sided number cube. They calculate probabilities by rolling number cubes, using spinners, and drawing marbles from a bag.	• An experiment is a situation involving chance that leads to results or outcomes. • An outcome is the result of a single trial of an experiment. • A sample space is the list of all possible outcomes of an experiment. • An event is one or a group of possible outcomes for a given situation. • A simple event is an event consisting of one outcome. • Probability is a measure of the likelihood that an event will occur. • The probability of an event can be determined by using the formula: Probability = the number of times an event occurs divided by the number of possible outcomes. • When the probability of an event is equal to 0 there is no chance that the event will occur. • When the probability of an event is equal to 1 there is certainty that the event will occur. • Complementary events are events that consist of the desired outcomes, and the remaining events that consist of all the undesired outcomes. • The sum of the probabilities of any two complementary events is 1.
2	**Give the Models a Chance** Probability Models	7.SP.7	2	Using tables and dot plots, students construct and interpret uniform and non-uniform probability models comparing experimental probabilities to theoretical probabilities.	• A probability model is a list of each possible outcome along with its probability. The sum of all the probabilities for the outcomes will always be 1. • A uniform probability model is a model in which all of the probabilities are equally likely to occur. • A non-uniform probability model is a model in which all of the probabilities are not equally likely to occur.
3	**Toss the Cup** Determining Experimental Probability of Simple Events	7.SP.6 7.SP.7.b 7.RP.3	2	Students conduct probability experiments and compute experimental probabilities. They compare the experimental and theoretical probabilities and use proportional reasoning to predict frequencies and calculate percent error.	• Experimental probability is the ratio of the number of times an event occurs to the total number of trials performed. • Theoretical probability is the ratio of the number of desired outcomes to the total possible outcomes. • Percent error is one way to measure the difference between experimental and theoretical probabilities.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**A Simulating Conversation** Simulating Simple Experiments	7.SP.6 7.SP.7.a	2	Students use simulations to explore the relationship between the theoretical probability and the experimental probability as the number of trials increases.	• A simulation is an experiment that models a real-life situation. When conducting a simulation, you must choose a model that has the same probability as the event. • A trial is a repetition of an experiment. Each time the experiment is repeated, it is called a trial. • Experimental probability of an event approaches the theoretical probability when the number of trials is large.
Learning Individually with MATHia or Skills Practice		7.SP.5 7.SP.6 7.SP.7	1	Students build probability models and determine probabilities of simple and disjoint events. They then use proportions to make predictions based on samples and theoretical probabilities. Students use results of probability experiments to make conjectures about theoretical probabilities. **MATHia Unit:** Introduction to Probability **MATHia Workspaces:** Determining Probabilities / Comparing Experimental and Theoretical Probability / Simulating Simple Events	

*Pacing listed in 45-minute days

Topic 2: Compound Probability

Students use arrays, lists, and tree diagrams to organize the possible outcomes of an experiment. They create probability models, calculate experimental and theoretical probabilities of events, and use proportional reasoning to determine percent error and to make predictions of expected numbers of outcomes. Then students learn about compound events that use the conjunctions "and" and "or." Students then design and conduct simulations for three compound probability problems.

Standards: 7.SP.6, 7.SP.7, 7.SP.8 **Pacing:** 7 Days

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Topic 3: Drawing Inferences

Students use random samples to collect representative data from a specified population. They use the results of the sample and proportional reasoning to estimate population parameters. Then students use data displays and measures of center and variation to compare populations and random samples to draw inferences about populations or to compare two populations.

Standards: 7.SP.1, 7.SP.2, 7.SP.3, 7.SP.4 **Pacing:** 9 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**We Want to Hear From You!** Collecting Random Samples	7.SP.1	2	Students review the statistical process and begin learning elements of rigorous data collection, include generating representative and random samples.	• A survey is a method of collecting information from a population or sample of a population. • A population is the entire set of items from which data can be selected. • A census is the collection of data from every member of a population. • The characteristic used to describe the population is called a parameter. • A statistic describes the sample from a population and can be used to make a prediction about a parameter. • A random sample is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected. • A sample generated randomly is more likely to be representative of the population than one that is not generated randomly. • Random number tables are used to generate random numbers when the population size is large.
2	**Tiles, Gumballs, and Pumpkins** Using Random Samples to Draw Inferences	7.SP.1 7.SP.2	2	Students use statistical information gathered from a sample along with proportional reasoning to determine a parameter for a population. They learn that statistics obtained from random samples are more likely to represent the parameter of the population than non-random samples.	• Statistics obtained from samples are more likely to represent the parameter of the population if the sample is randomly chosen. • Statistics are used to estimate parameters. • Proportional reasoning can be used with statistics to estimate parameters. • Percent error can be used as a measure of the variation between a statistic and a parameter.
3	**Dark or Spicy?** Comparing Two Populations	7.SP.3	2	Students calculate the measures of center and measures of variability for two different populations. They plot the data and compare the measures of center with respect to the measure of variation.	• The mean and the spread of data for two populations can be determined from a graph or dot plot. • When the centers for two populations are equivalent, the mean absolute deviation can show the actual differences in variability between the two data sets of the two populations.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Finding Your Spot to Live** Using Random Samples from Two Populations to Draw Conclusions	7.SP.3 7.SP.4	2	Students use random samples to draw conclusions about two populations. They use means and mean absolute deviations and medians and interquartile ranges.	• Measures of center for samples from two populations are compared. • Graphical displays such as stem-and-leaf plots and box-and-whisker plots are used to determine the characteristics of two populations.
Learning Individually with MATHia or Skills Practice		7.SP.3	1	Students compare the characteristics of data displays, specifying which numerical characteristics can be determined from each display. They then use data displays to compare populations by determining the visual overlap and describing the difference between the measures of centers in terms of measures of variability. **MATHia Unit:** Numerical Data Displays Comparisons **MATHia Workspaces:** Using Statistics to Draw Inferences About a Population / Comparing Characteristics of Data Displays / Comparing Populations Using Data Displays	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

5 Constructing and Measuring

Pacing: 19 Days

Topic 1: Angles and Triangles

Students learn about formal constructions and use construction tools to duplicate segments and angles. Students explore and use different pairs of angles including supplementary angles, complementary angles, vertical angles, and adjacent angles. Finally, students use both patty paper and formal construction tools to determine if given information defines a unique triangle, multiple triangles, or no triangles.

Standards: 7.G.2, 7.G.5 **Pacing:** 9 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Here's Lookin' at Euclid** Geometric Constructions	7.G.2	2	Students are introduced to geometry and geometric constructions. Measuring tools are distinguished from construction tools, and the concepts of sketch, draw, and construct are differentiated. Students will learn how to properly draw, sketch, and name each of the essential building blocks of geometry. They use a compass to construct circles and arcs and to duplicate line segments and angles using only construction tools.	• To sketch a geometric figure is to create it without the use of tools, to draw a geometric figure is to create is using any tools, and to construct a geometric figures is to create it using only a compass and straightedge. • Three undefined terms in geometry are point, line, and plane. They cannot be defined, only described; a point is described as a location in space, a line is described as a straight continuous arrangement of an infinite number of points, and a plane is described as a flat surface. • A line segment is a portion of a line that includes two points and all the points between those two points. The endpoints are the points where the line segment begins and ends.
2	**Special Delivery** Special Angle Relationships	7.G.5	2	Students use protractors and patty paper to explore special angle pairs formed when two lines intersect. They use the definitions and write and solve equations about special angle pairs.	• Two angles are supplementary if the sum of their angle measures is equal to 180°. • Two angles are complementary if the sum of their angle measures is equal to 90°. • Two lines, line segments, or rays are perpendicular if they intersect to form 90° angles. • Patty paper and protractors are tools used to explore line and angle relationships. • Adjacent angles are two angles that share a common vertex and share a common side. • A linear pair of angles is formed by two adjacent angles with non-common sides that form a line. • The angles in a linear pair are supplementary. • Vertical angles are two nonadjacent angles that are formed by two intersecting lines. • Vertical angles have the same measure, or are congruent.
3	**Consider Every Side** Constructing Triangles Given Sides	7.G.2	2	Students use pasta, patty paper, and construction tools to determine the number of segments and the conditions needed to construct a unique triangle, more than one triangle, or no triangles.	• Constructing a triangle given the length of two sides does not result in the construction of a unique triangle. • Constructing a triangle given the length of three segments, such that the sum of two segment lengths is greater than the third length, results in the construction of a unique triangle.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Unique or Not?** Constructing Triangles Given Angles	7.G.2	2	Students use patty paper and construction tools to determine what information is needed to construct a unique triangle, more than one triangle, or no triangles when given at least one angle.	• Constructing a triangle given the measure of three angles does not result in the construction of a unique triangle. • Constructing a triangle given the measure of two angles and the length of one side does not result in the construction of a unique triangle. • An included angle is the angle whose sides are made up of the specified sides of a triangle. • An included side is the side between two specified angles of a triangle. • Constructing a triangle given the length of two sides and the measure of the included angle results in the construction of a unique triangle. • Constructing a triangle given the measure of two angles and the length of the included side results in the construction of a unique triangle.
Learning Individually with MATHia or Skills Practice		7.G.5	1	Students measure angles and determine angle sums. They then identify complementary, supplementary, and vertical angles. Students write and solve equations to solve for unknown angle measures. **MATHia Unit:** Angle Properties **MATHia Workspaces:** Calculating Angles / Classifying Angles and Determining Unknown Measures	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Topic 2: Three-Dimensional Figures

Students create and describe cross-sections of right rectangular prisms and right rectangular pyramids. They determine the areas of regular polygons through decomposition. Then, they calculate the volumes and surface areas of right prisms and pyramids.

Standards: 7.G.3, 7.G.6 **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Slicing and Dicing** Cross-Sections of Rectangular Prisms	7.G.3	2	Students explore cross-sections of cubes and general right rectangular prisms. They determine how to make each of six different cross-section results with each solid.	• A cross-section of a solid is the two-dimensional figure formed by the intersection of a plane and a solid when a plane passes through the solid. • A right rectangular prism has bases that are squares and lateral faces that are rectangles. • The possible cross-sections formed when any right rectangular prism is sliced by a plane are a square, a rectangle that is not a square, a triangle, a pentagon, a hexagon, and a parallelogram that is not a rectangle. • Cross-sections are formed when a plane slices through any right rectangular prism. Different cross-sections are formed based on where the plane slices through the right rectangular prism.
2	**Dissecting a Pyramid** Cross-Sections of Rectangular Pyramids	7.G.3	1	Students explore cross-sections of right rectangular pyramids. They determine how to make three different cross-section results with the pyramids.	• A right rectangular pyramid has a rectangular base and four triangular lateral faces, with the height perpendicular to the base. • Cross-sections are formed when a plane slices through a right rectangular pyramid. Different cross-sections are formed based on where the plane slices through the right rectangular pyramid. • The possible cross-sections formed when a right rectangular pyramid is sliced by a plane are a triangle, a rectangle that is not a square, and a trapezoid.
3	**Hey, Mister, Got Some Bird Seed?** Volume of Pyramids	7.G.6	2	Students use nets of an open rectangular prism and an open rectangular pyramid with congruent bases and heights to investigate the volume of pyramids. They use the volume formulas to solve problems involving prisms and pyramids. Students then investigate the effect that doubling and tripling dimensions of prisms and pyramids has on their volumes.	• A pyramid is a polyhedron with one base and the same number of triangular faces as there are sides of the base. The triangular faces are called lateral faces. • A rectangular pyramid is a pyramid that has a rectangle as its base. • A triangular pyramid is a pyramid that has a triangle as its base.

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**The Sound of Surface Area** Surface Area of Pyramids	7.G.6	2	Students compare two different pieces of acoustical foam — one that is made up of square pyramids and one that is made up of triangular prisms — and determine their surface areas. They use formulas to calculate surface area and volumes of rectangular and triangular prisms and pyramids.	• A prism is a polyhedron with two parallel and congruent faces called bases. All other faces are parallelograms and are called lateral faces. • A rectangular prism is a prism that has rectangles as its bases. • A triangular prism is a prism that has triangles as its bases. • A pyramid is a polyhedron with one base and the same number of triangular faces as there are sides of the base. The triangular faces are called lateral faces. • A rectangular pyramid is a pyramid that has a rectangle as its base. • A triangular pyramid is a pyramid that has a triangle as its base.
5	**More Than Four Sides of the Story** Volume and Surface Area of Prisms and Pyramids	7.G.6	2	Students use the strategies for calculating the volumes and surface areas of right rectangular prisms and pyramids to calculate the volumes and surface areas of prisms and pyramids with non-rectangular bases. They also develop a strategy to calculate the areas of regular polygons.	• The volume of any prism can be calculated by the formula $V = Bh$, where B is the area of the base and h is the height of the prism. • The volume of any pyramid can be calculated by the formula $V = (1/3)Bh$, where B is the area of the base and h is the height of the pyramid. • The surface area of any geometric solid is the sum of the areas of the surfaces of the solid. • A regular polygon is a polygon with congruent sides and congruent angles. • A regular n-gon can be decomposed into n congruent triangles. • The area of a regular n-gon can be calculated by determining the area of one of the n congruent triangles and multiplying by n. • Polygons and solids can be composed to create additional figures whose areas, surface areas, and volumes can be determined.
	Learning Individually with MATHia or Skills Practice	7.G.6	1	Students calculate the volume of pyramids in mathematical and real-world contexts. **MATHia Unit:** Three-Dimensional Figures **MATHia Workspace:** Visualizing Cross Sections of Three-Dimensional Shapes **MATHia Unit:** Volume of Prisms and Pyramids **MATHia Workspaces:** Understanding Volume Formulas for Right Prisms / Using Volume of Right Prisms / Calculating Volume of Pyramids / Using Volume of Pyramids	

Total Days: 138

Learning Together: 103
Learning Individually : 35

*Pacing listed in 45-minute days